Math 1

Spring Final Exam Review

Interval Notation. For continuous functions, we use interval notation to describe domain and range as well as where the function is increasing or decreasing. We use brackets to include endpoints and parentheses to exclude endpoints. Note that $-\infty$ and ∞ are *always* excluded. For intervals of increasing and decreasing, we work with the values of x, and we *never* use brackets.

Special Sets.

- The symbol $\mathbb{N}$ stands for the set of all natural numbers: $\{1, 2, 3, 4, 5, \dots\}$
- The symbol $\mathbb{Z}$ stands for the set of all integers: $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ We use $\mathbb{Z}$ for this set because of the German word for this set, which is "Zahlen" (pronounced "tsah-lenn").
- The symbol $\mathbb{R}$ stands for the set of all real numbers. Thus, $\mathbb{R} = (-\infty, \infty)$.

Graphical Displays.

- Make the displays as neat as possible. You may want to use a straightedge and/or graph paper.
- Every display *must* have a title that relates to the data being displayed.
- If a graphical display has more than one axis, label the axes appropriately. The general labels x and y are *not* appropriate for data.

Averages of Data.

- Do *not* say "the average". There is more than one average.
- The mode of a set of data is the piece of data that occurs the most frequently. If there is a two-way tie, then both of them are modes (and the data set is "bimodal"). In general, if there is more than a two-way tie, then the set of data has no mode.
- The mean of a set of measurement data is the sum of all of the data divided by the number of pieces of data.
- For a set of measurement data with an odd amount of pieces of data, the median is the piece of data in the very middle when the data are in numerical order.
- For a set of measurement data with an even amount of pieces of data, the median is the mean of the two pieces of data in the very middle when the data are in numerical order.

Numbers for Box and Whiskers.

- The minimum of a set of measurement data is the smallest piece of data.
- The maximum of a set of measurement data is the largest piece of data.
- Q_1 is the median of the half of the data that are below the median. (For an odd amount of data, the lower half excludes the median.)
- Q_3 is the median of the half of the data that are above the median. (For an odd amount of data, the upper half excludes the median.)

What r Means. If the diagnostics are on, the graphing calculator will give a value of r for a linear regression. This is what r tells you:

- If $r > 0$, then, in general, as the value of one variable increases, the other variable also increases. Thus, the slope of the line of least squares is positive.
- If $r < 0$, then, in general, as the value of one variable increases, the other variable decreases. Thus, the slope of the line of least squares is negative.
- The further r is from 0, the better the line approximates the data.
- If $r^2 = 1$, then the line of least squares passes through *all* of the data points.

Forms of Equations of Lines.

- If a line has slope m and passes through (h, k), then its point-slope form is $y - k = m(x - h)$.
- If a line has slope m and passes through $(0, b)$, then its slope-intercept form is $y = mx + b$.

Parallel and Perpendicular Lines.

- Parallel lines have the same slope.
- If two lines that are neither horizontal nor vertical are perpendicular to each other, then their slopes are opposite reciprocals of each other.
- In order for a line to reflect one point onto another, it must be the perpendicular bisector of the line segment whose endpoints are the two given points.

Distance Between Points. The distance between (x_1, y_1) and (x_2, y_2) is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Instead of memorizing this formula, it is *much* easier just to use the Pythagorean Theorem.

Equation of a Circle. The equation of a circle with center (h, k) and radius r is

$$(x - h)^2 + (y - k)^2 = r^2.$$

Note the similarity of this formula to the Pythagorean Theorem and the Distance Formula.

Vertical Shift. A vertcial shift of f is of the form

$$g(x) = f(x) + k$$

for some $k \in \mathbb{R}$. If $k > 0$, then g is a shift of f up k units. If $k < 0$, then g is a shift of f down $|k|$ units.

Horizontal Shift. A horizontal shift of f is of the form

$$g(x) = f(x + k)$$

for some $k \in \mathbb{R}$. If $k > 0$, then g is a shift of f left k units. If $k < 0$, then g is a shift of f right $|k|$ units.

Linear Combinations of Vectors. If $\vec{u} = \langle a, b\rangle$ and $\vec{v} = \langle x, y\rangle$, then

$$r\vec{u} + s\vec{v} = \langle ra + sx, rb + sy\rangle.$$

Magnitude. If $\vec{v} = \langle x, y \rangle$, then the magnitude of $\vec{v}$ is

$$\|\vec{v}\| = \sqrt{x^2 + y^2}.$$

Note again the similarity to the Pythagorean Theorem

Radical Issues. Beware of the following:

$$\sqrt{m} + \sqrt{n} \neq \sqrt{m+n}$$

Magnitude Issues. Beware of the following:

$$\|\vec{u} + \vec{v}\| \neq \|\vec{u}\| + \|\vec{v}\|$$

Determinant. The determinant of $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is $ad - bc$.

Area of a Parallelogram. If $\langle a, c \rangle$ and $\langle b, d \rangle$ are distinct vectors that emanate from the same vertex of a parallelogram, then the area of the parallelogram is $|ad - bc|$.

Inverse of a Matrix. A matrix has an inverse if and only if its determinant is nonzero. The following formula yields the inverse of a 2×2 matrix:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$